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Scale Spaces for Cognitive Systems: A Position Paper

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1 Scale Spaces and Cognitive Systems

Images, like the real world, contain information at many different spatial (and temporal) scales. In image processing and computer vision, the *scale space* is a classic approach for establishing an explicit representation of the multi-scale nature of image content. Introduced by Iijima in 1962 [4] and extensively studied then in Japan [14], it has received attention in western countries since Witkin's work [15]. Scale-space theory is an important building block for models of *cognitive systems* as it puts fundamental concepts of the perceptual acquisition of knowledge into a mathematical framework [8, 9].

The Idea behind Scale Spaces. Relevant image features (such as the edge of a forest, the trunk of a tree, or the edges of a leaf) in any given image of a scene can be found over a certain – a priori unknown – range of spatial scales. Thus, an image description that emphasises the hierarchy of scales will be very useful in understanding image content. Basically, an image f is represented as an ordered family of increasingly smoothed versions of the original image. Initially, details observable only at a fine scale (e.g. distant leaves) are removed while large-scale structures are kept. As the amount of smoothing is continously increased, more and more detail will be removed. Thus, important image structures are ordered by their spatial scale.

Let us denote now the family of scale spaces for an input image f by $\{T_t f : t \geq 0\}$, where the parameter t refers to the scale. Then the scale-space requirements that reflect the need of a hierarchical image representation can be described via three classes of axioms [2]: *(i)* axioms related to the underlying *structure of the scale space* such as e.g. the semi-group property $T_{t+s}f = T_t\,(T_s f)\ \forall t,s \geq 0$; *(ii)* axioms describing *simplification properties* such as e.g. the causality principle which states

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that structures found at some scale $t^* > 0$ can always be traced back to finer scales $0 \leq t < t^*$; *(iii)* axioms describing *invariances* such as e.g. translation or rotation invariance.

The oldest and best-understood scale space [4] is the *Gaussian scale space*. It is generated by the convolution of an input image with Gaussian kernels K_σ of increasing variance σ^2, i.e. $T_t f := K_{\sqrt{2t}} * f$. Equivalently, it can be stated by a solution of the *partial differential equation* (PDE) of linear diffusion $u_t = \Delta u$ where Δ is the Laplace operator, with the input image as initial condition. Here, the scale space parameter t can be interpreted as evolution time. This description immediately gives rise to a variety of generalisations that are based on different PDEs and thus capitalise on the high flexibility of PDEs as a modelling tool. Examples are nonlinear diffusion scale spaces [13] as well as so-called morphological scale spaces [12]. Since PDEs arise also as Euler-Lagrange equations, this links scale spaces directly to variational methods in continuous-scale optimisation [10].

Relation to Visual Perception and Cognitive Systems. A large body of research is dedicated to examining the relationship between mathematical aspects of scale spaces and aspects of the human visual system (see e.g. [7, 11]). For example, Koenderink has shown that the early stages of the visual cortex seem to use a form of multiscale differential geometry to represent images [5]. Likewise, the structure of receptive fields in the retina and of the early visual cortex can be modelled using a Gaussian scale-space. Moreover, the precise layout of receptive field sizes on the retina is exactly what is needed to create a scale-space representation. More intriguingly, the usually ignored fact that there are many more connections feeding from higher visual areas down into lower visual areas than feed upwards from lower to higher areas is consistent with the needs of scale-space construction [11], and has made several predictions (e.g. the temporal modification of receptive field structure) that have since been verified [1].

Directions for Future Research. Recent interest in scale-space research goes in numerous directions, see e.g. the conference volume [3]. One field of investigation is the integration of different colour models related to human colour perception. Another trend is the extension of scale-space models by non-local concepts that allow to exploit similarities between structures occurring at different locations in an image. Other works aim at incorporating representations of texture information into scale-space frameworks. On the application side, there are new attempts to the analysis of audio signals by scale-space methods [6].

We believe that the potential of the scale space viewpoint has still not been exhausted. It can be considered a pivotal concept for the theoretical understanding of different approaches to signal and image processing. Its ability to provide mathematical stringent models for processes of human perception together with the versatility of the underlying mathematical concepts lets expect that it can be used for even more detailed modelling of cognitive processes. On this basis, it can guide the development of human-computer interaction processes.

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